TRUSS ITN

Workshop

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TRUSS
Training in Reducing Uncertainty in Structural Safety

Sensitivity of SHM Sensors to Bridge Stiffness

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Sensitivity of SHM Sensors to Bridge Stiffness

Theorem of Virtual Work → $$\delta = \int \frac{MM_u}{EI} dx$$ → **valid for statics with sensors on the bridge**

Matrix representation of Theorem → $$\{\delta\}_{t\times 1} = [\mathrm{P}]_{t\times n}\{\mathrm{J}\}_{n\times 1}$$ → **where P is the matrix representing MM_u values and J is vector of $\frac{1}{EI}$ components (reciprocal of stiffness)**

$$EI_n = \frac{1}{J_n}$$

Sensitivity of SHM Sensors to Bridge Stiffness

Sensitivity

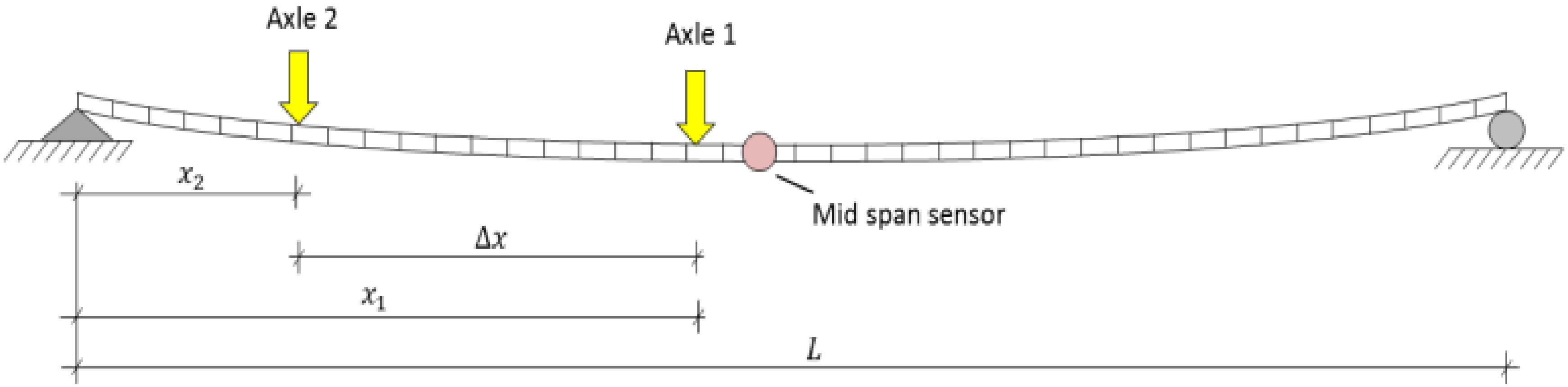
$$S(i,j) = \frac{\partial u_i}{\partial J_j}$$

$S(i,j)$**: Sensitivity of the deflection respect to the reciprocal of the flexural stiffness**

J_j**: reciprocal of the flexural stiffness at element** j

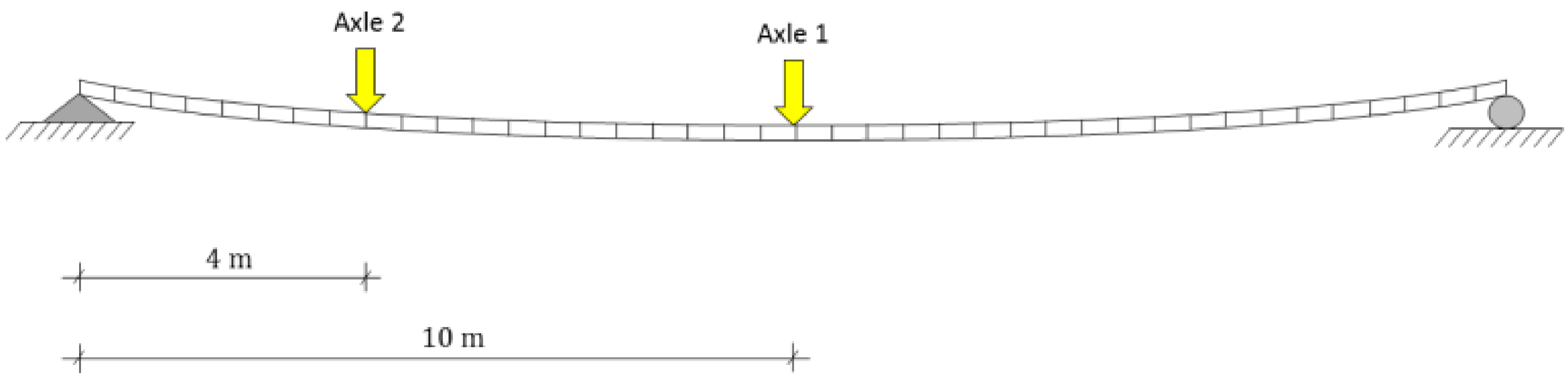
Sensitivity of SHM Sensors to Bridge Stiffness

Sensor Installation at mid span

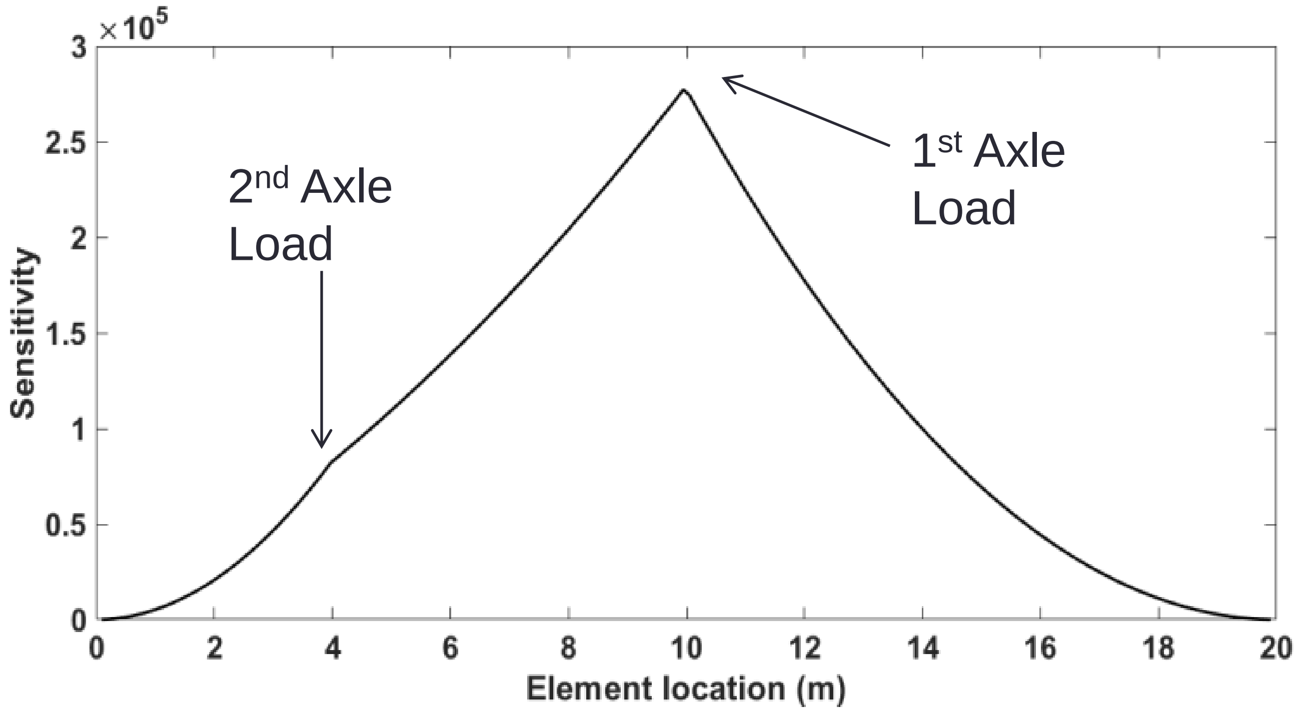


Sensitivity of SHM Sensors to Bridge Stiffness

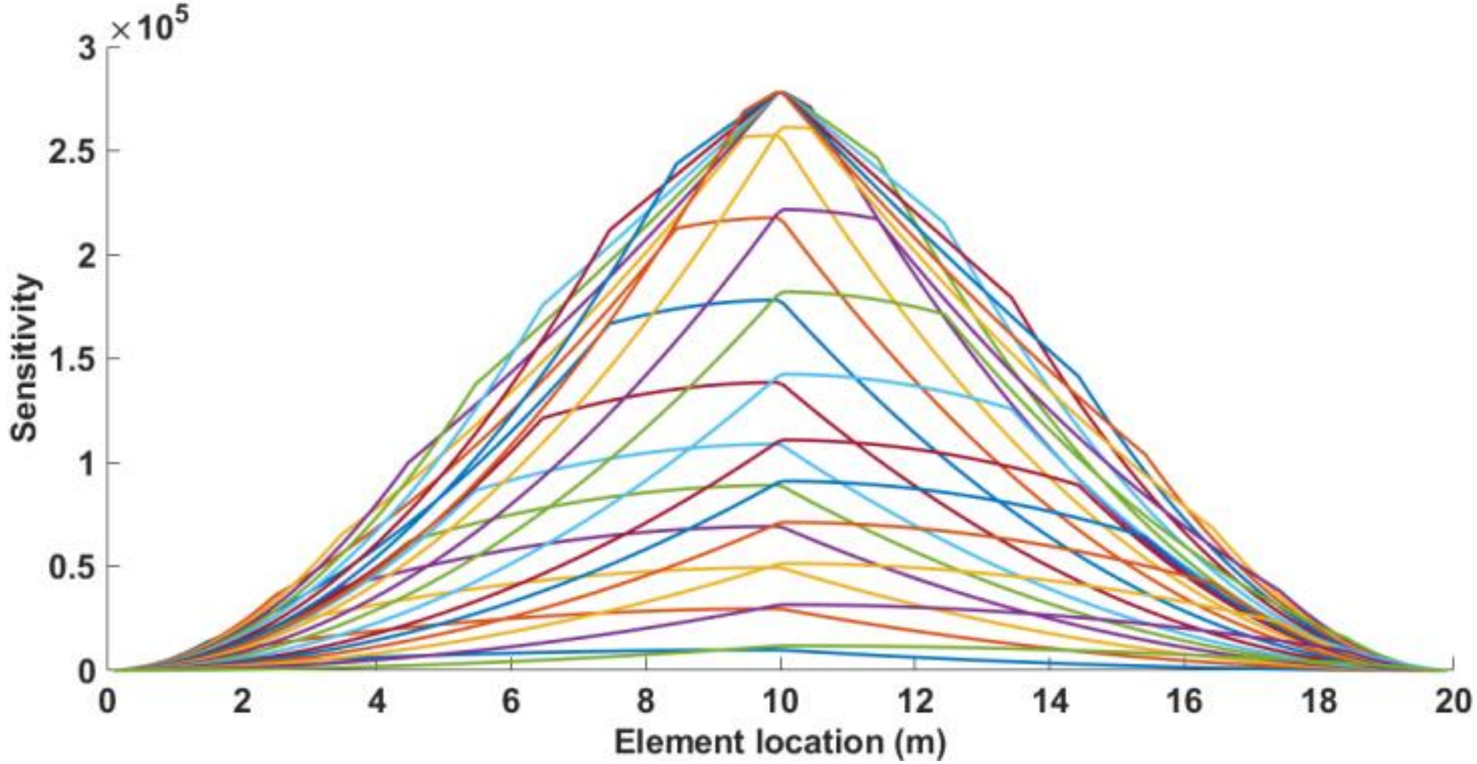
Situation Analysed



Sensitivity of SHM Sensors to Bridge Stiffness



Sensitivity of SHM Sensors to Bridge Stiffness



Envelope of the sensitivities

Calculating Bridge Stiffness from deflection measurements

$$\{\delta\}_{t\times 1} = [P]_{t\times n}\{J\}_{n\times 1}$$

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{Bmatrix} = \begin{bmatrix} P_{1A} & P_{1B} & P_{1C} \\ P_{2A} & P_{2B} & P_{2C} \\ P_{3A} & P_{3B} & P_{3C} \\ P_{4A} & P_{4B} & P_{4C} \\ P_{5A} & P_{5B} & P_{5C} \end{bmatrix} \times \begin{Bmatrix} J_A \\ J_B \\ J_C \end{Bmatrix} \Rightarrow b = Ax$$

A **IS A NON SQUARED MATRIX AND NOT INVERTIBLE**

We can solve to find the stiffnesses only if the matrix is square, i.e., only if the number of unknown J’s equals the number of measurements

Calculating Bridge Stiffness from deflection measurements

$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_3\\ \delta_4\\ \delta_5\end{Bmatrix} = \begin{bmatrix} P_{1A} & P_{1B} & P_{1C}\\ P_{2A} & P_{2B} & P_{2C}\\ P_{3A} & P_{3B} & P_{3C}\\ P_{4A} & P_{4B} & P_{4C}\\ P_{5A} & P_{5B} & P_{5C}\end{bmatrix} \times \begin{Bmatrix} J_A\\ J_B\\ J_C\end{Bmatrix}$$

The system of equations is also ill conditioned, i.e., even when the matrix is square, accuracy is very sensitive to noise in the measurement

Calculating Bridge Stiffness from deflection measurements

Two problems:
(i) Matrix not square,
(ii) System is ill-conditioned

Moses algorithm
+
Penalty function

Moses algorithm minimises the differences between measured deflections and the theoretical response. It solves the problem of the non-square P matrix

Penalty function ensures that there is no sudden change of neighbouring stiffness values, avoiding, for example, negatives. It is based on a Blackman window. It solves the ill conditioning problem.

Calculating Bridge Stiffness from deflection measurements

Moses algorithm

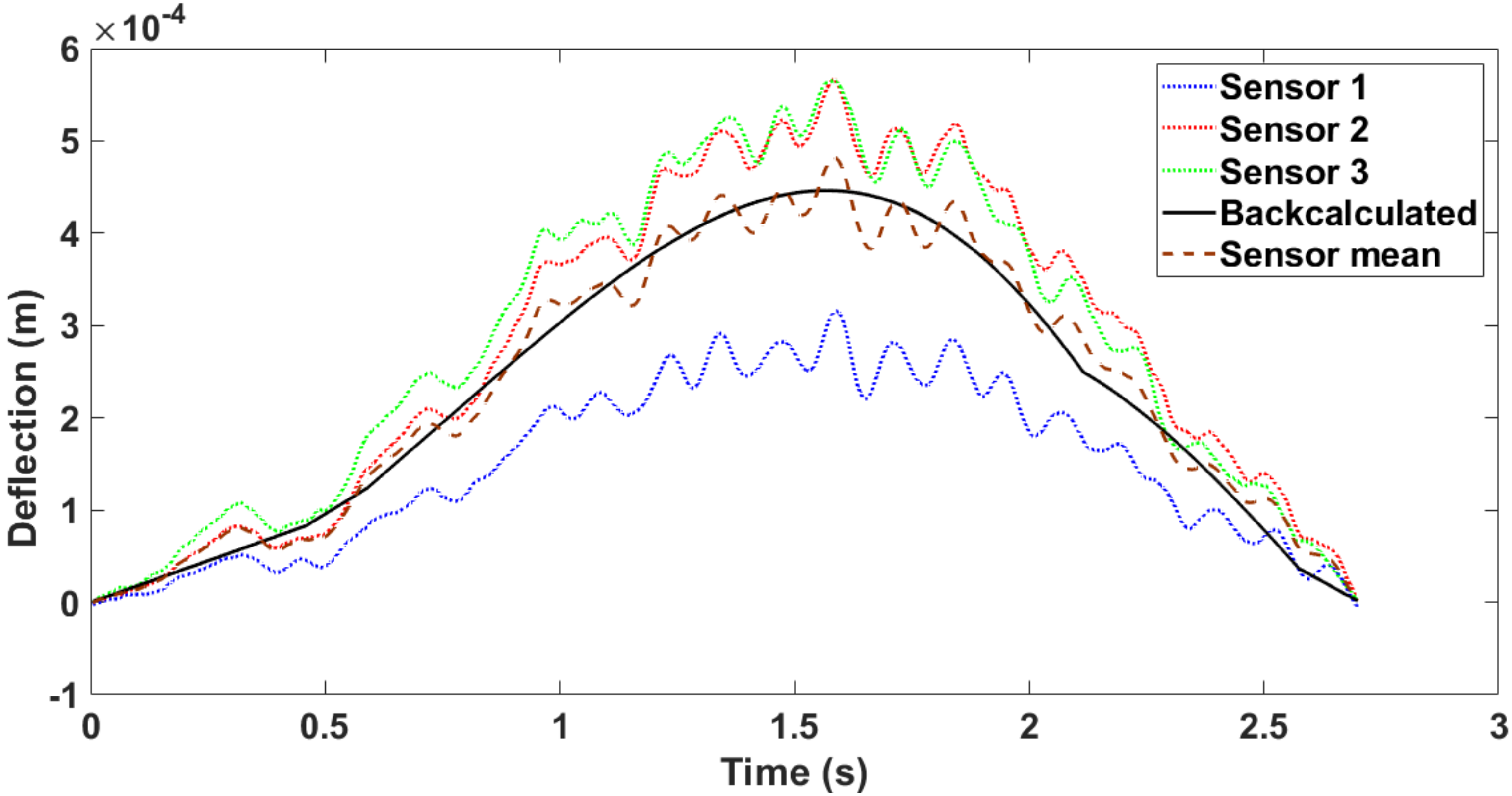
$$E = min\left[(\{\delta\} - [P]\{J\})^2\right]$$

Penalty Function

$$\sum_{i=1}^{n-1} k\,(-c_1 J_{i-b} - \cdots - c_b J_{i-1} + J_i - c_{b+1} J_{i+1} - \cdots - c_{2b} J_{i+b})^2$$

Where k is a set constant, b is the number of points used in the filter each side and the sum of constants c_1 to c_{2b} equals 1

Calculating Bridge Stiffness from deflection measurements



Conclusions

- Deflection sensitivity to stiffness is greater at mid-span.
- Deflection data with loading conditions closer to mid-span should improve stiffness calculations.
- Moses and penalty function can potentially calculate stiffness decreasing the influence of noise.

Thanks for your attention

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